Quantitative Reasoning: The Next “Across the Curriculum” Movement

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We live in an age where vast amounts of information can be accessed on the Internet. Much of this information is quantitative in nature and students (and adults) must be equipped to analyze the information as they sift through the data to make decisions in their everyday lives. And, as “big data” analyses move from pure research applications to business, education, health, and government settings where our graduates will be working, this imperative becomes more critical. Even in our own institutions of higher education, we are crunching large data sets of student information to monitor and predict student performance and success. All of these situations require strong quantitative reasoning skills.

WHAT IS QUANTITATIVE REASONING?
Quantitative reasoning. Quantitative literacy. Quantitative fluency. Numeracy. These are often-used terms when discussing a key learning outcome for undergraduate education. Here are a few high-profile examples of calls for prioritizing such quantitative skills:

- Quantitative literacy is one of the LEAP (Liberal Education for America’s Promise) Essential Learning Outcomes (ELOs) developed by the Association of American Colleges & Universities (AAC&U), one of a number of practical intellectual skills, including inquiry and analysis, critical and creative thinking, written and oral communication, information literacy and teamwork, and problem solving.
- The Lumina Foundation’s Degree Qualifications Profile (DQP) calls this skill quantitative fluency and places it, like LEAP, among several important intellectual skills all students should attain, including analytic inquiry, information literacy, engaging diverse perspectives, and communication fluency.
- The Western Association of Schools and Colleges (WASC) Senior College and University Commission has recently shifted its focus to five core competencies—writing, oral communication, quantitative reasoning, critical thinking, and information literacy—in its revised institutional review process.

The ability to think quantitatively clearly plays a central role in undergraduate education. But what do terms like quantitative reasoning, quantitative literacy, and quantitative fluency really mean for student learning, the curriculum, program development, faculty development, or accreditation? Why is it such an important outcome? How do we teach and measure it? Who is responsible for ensuring that students achieve this competency?

By one definition, quantitative reasoning (QR) is the application of basic mathematics skills, such as algebra, to the analysis and interpretation of real-world quantitative information in the context of a discipline or an interdisciplinary problem to draw conclusions that are relevant to students in their daily lives. It is not just mathematics. Carleton College, for example, views QR as “the habit of mind to consider the power and limitations of quantitative evidence in the evaluation, construction, and communication of arguments in public, professional, and personal life.” The term numeracy is also used in conjunction with these skills.

Ultimately, QR requires students to think critically and apply basic mathematics and statistics skills to interpret data, draw conclusions, and solve problems within a disciplinary or interdisciplinary context (fig. 1). Indeed, it requires the kind of mathematical and statistical skills that should be developed in high school, so all college students should have the basic skills required to achieve this broader, more ambitious college-level outcome. It is a competency of integration and application, both of which are intellectual capacities up near the top of the cognitive skills taxonomy originally described by Bloom (1956). Assignments that develop QR can also elicit demonstration of achievement of other key outcomes like writing and/or oral communication as well as
Examples of QR in everyday life abound and can be drawn upon to teach QR in the context of virtually any discipline. They can be found in areas such as health, economics, politics, science, engineering, social science, and even the arts. For example, virtually all parents face the vaccination question early in the life of their children. Parents might ask questions like, “What are the risks associated with vaccinating my child? What are the benefits?” In order to answer these questions, they must take into account quantitative information, such as disease occurrence rates in populations over time, or numbers of cases of complications with certain vaccine preparations. In today’s information age, the Internet is the most readily available source of information, so students (and adults) must be able to discern reliable versus non-reliable sources.

Returning to our vaccination example, there is rampant misinformation online about a connection between autism and vaccinations that must be recognized as such when parents formulate their decisions. Making judgments based on political polling data, understanding the national debt, interpreting nutrition facts, evaluating medical treatment or screening options, making investment decisions, and even purchasing decisions—all are everyday challenges that require us to use QR skills. However, according to a 2003 survey by the National Assessment of Adult Literacy, only 13 percent of adults are deemed proficient in quantitative literacy; 33 percent perform at intermediate levels, 33 percent at basic levels, and 22 percent are below basic.

Larger societal issues, such as climate change, also require the application of QR skills—and the closing of a widening gap between those who have these skills and those who do not. Issues like these are politically contentious, beyond the practical implications for everyday life and decision making (should I buy a hybrid car? Should I buy carbon credits?). The “hockey stick” graph of rising CO2 levels made worldwide news as politicians debated the science behind climate change, or global warming as it was known in the past decade.

Jon D. Miller is a political scientist at University of Michigan who has been studying the civic scientific literacy of US adults. In surveys that ask basic factual scientific questions, he finds that less than 30 percent are scientifically literate (Miller 2010). Anthony Carnevale, director of research at the Center on Education and the Workforce at Georgetown University, argues that “the remedy for the widening gulf between those who are literate in mathematics and science and those who are not is democratization—making mathematics and science more accessible and responsive...to the needs of all citizens” (Steen 2004, 65). One way to achieve this literacy may be through a more intensive focus on quantitative reasoning in college. There are implications for all levels of education, preschool through college, but our focus here is on the undergraduate curriculum.

One of the primary misconceptions regarding QR is that it is already taught in mathematics classes. However, QR is different from math. QR utilizes basic mathematics skills in the service of carrying out
complex reasoning and decision-making processes. It is less about the how to perform the calculation and more about the meaning of the calculation results. Figure 2 contrasts math and QR to highlight the differences between them (Steen 2004).

A recent paper by Rocconi and colleagues (2013) reports that students in STEM fields are more engaged in QR-related activities than those in non-STEM fields, with students in education and the humanities showing the least engagement. This may not be surprising, but it is illuminating, given that QR skills are important for all students. It is easy to assume that the responsibility for QR should rest with the mathematics portion of general education or mathematics faculty. But experts argue that QR goes beyond basic math skills, and that most math courses don’t teach QR skills. There is a disciplinary context to the deep demonstration of QR skills by students that can most likely only be achieved by repeated exposure across the curriculum, along with culminating assessment in the major or a capstone experience. Faculty in mathematics departments may be best suited to take a leadership role in leading a campus-wide effort, but that effort must include faculty in other disciplines to have the broadest impact.

**HOW DO WE GET THERE?**
A 2001 study by the Mathematical Association of America summed up the challenges:

1. Most higher education students graduated without sufficient QR skills
2. Faculty in all disciplines needed professional development support to enhance QR in their courses
3. QR was not part of assessment activity
4. Education policy leaders were insufficiently aware of the increasing need for QR

While this study is more than a decade old, we may not be much further along today. QR is a complex outcome that requires immediate attention from faculty across the disciplines. Many institutions have embraced the core competencies of writing and communication, but far fewer attend to this equally critical outcome.

In addition, there are special difficulties in reaching students. As Hughes-Hallett (2001) notes, they find it hard, especially when QR is taught in the context of the disciplines. She describes results from a study where students were given a quantitative problem to solve in the abstract and then in the context of a scientific problem. No scientific understanding was required to solve the problem, but students had trouble with the contextualized problem, in part because their perceptions of science or science phobia interfered. Other challenges might be related to creating awareness and buy-in across the campus for establishing and measuring QR outcomes. Campuses are already measuring many outcomes as accreditors ask for more specific and deliberate outcomes assessment (such as WASC’s new required attention to five core competencies discussed above). Many campuses have yet to define this outcome. Thus, an initial hurdle may be just starting the conversation about what QR means.

**LEARNING OUTCOMES FOR QUANTITATIVE REASONING**
As with any core competency or higher-order intellectual skill, using the “backward design” process (Wiggins and McTighe 1998) to define the desired outcomes and create appropriate assessments before designing learning experiences for students is useful. The outcomes may be simple or complex, depending on the focus or the locus of QR in the curriculum (i.e., general education or the major or some other institution-level requirement). These outcomes may include the kinds of math skills required, the types of data students should be able to interpret, the methods to be used for problem solving, the desired results of the application of these skills, and the ability to clearly communicate findings. Other outcomes may include student attitudes toward accomplishing these kinds of tasks, or ability to make connections to learning in the major or across the curriculum. Steen (2004b, 24) argues that there are three essential components to QR: (1) engagement with the real world (which may set it apart from traditional mathematics), (2) ability to apply quantitative thinking to unfamiliar contexts, and (3) adaptable reasoning, which is the ability to make judgments even in the “absence of sufficient information or in the face of inconsistent evidence.” How often in the real world do we have all the information we need to make a solid judgment? Rarely. Thus, we should be preparing our students to grapple with that kind of uncertainty.

Several universities have already developed outcomes for QR. One example of a comprehensive set of outcomes for graduating seniors at the University of

![FIGURE 2. MATH VERSUS QUANTITATIVE REASONING](image-url)
Virginia is shown in fig. 3. These outcomes are quite extensive but traverse the terrain of basic understanding of quantitative information and processes, using QR methods, communicating quantitative information, and evaluating quantitative information.

Another example is the DQP, which defines quantitative fluency, in terms of both what students should be able to do and at what level of skill or performance.

At the associate level, the student
- Presents accurate calculations and symbolic operations, and explains how such calculations and operations are used in either his or her specific field of study or in interpreting social and economic trends.

At the bachelor’s level, the student
- Constructs, as appropriate to his or her major field (or another field), accurate and relevant calculations, estimates, risk analyses, or quantitative evaluations of public information and presents them in papers, projects, or multimedia events.

At the master’s level:
- Students who are not seeking a degree in a quantitatively based field employ and apply mathematical, formal logic, and/or statistical tools to problems appropriate to their field in a project, paper, or performance.
- Students seeking a degree in a quantitative-based or quantitatively relevant field articulate and/or undertake multiple appropriate applications of quantitative methods, concepts, and theories within their field of study.

A key component of WASC’s new core competency requirement is that colleges and universities establish standards of performance that students should reach at or near graduation. This means that standards regarding how well or at what level students will be expected to perform must be established.

**ASSESSMENT**

Many different approaches to assessing QR have been developed, ranging from direct to indirect measures of learning. Available tools include ready-to-use instruments and rubrics as well as survey and interview questions that assess attitudes toward mathematics in real-world contexts. Examples are available on the national organizations’ websites described in the next section, but I will describe three specific tools below.

The Center for Assessment and Research Studies at James Madison University has developed the Quantitative Reasoning Test (Sundre 2008). This instrument has been administered at over fifty universities to more than 20,000 students.

**FIGURE 3. UNIVERSITY OF VIRGINIA QUANTITATIVE REASONING OUTCOMES**

A graduating fourth-year undergraduate at the University of Virginia will be able to

1. Interpret mathematical models such as formulas, graphs, tables, and schematics, and draw inferences from them.
2. Communicate mathematical information symbolically, visually, numerically, and verbally.
3. Use arithmetical, algebraic, and geometric methods to solve problems.
4. Estimate and check answers to mathematical problems in order to determine reasonableness.
5. Solve word problems using quantitative techniques and interpret the results.
6. Apply mathematical/statistical techniques and logical reasoning to produce predictions, identify optima, and make inferences based on a given set of data or quantitative information.
7. Judge the soundness and accuracy of conclusions derived from quantitative information, recognizing that mathematical and statistical methods have limits and discriminating between association and causation.
8. Solve multistep problems.
9. Apply statistics to evaluate claims and current literature.
10. Demonstrate an understanding of the fundamental issues of statistical inference, including measurement and sampling.

With funding from the National Science Foundation, Eric Gaze and colleagues have developed another tool, the quantitative reasoning and literacy test (QLRA) for measuring students’ QR skill levels. This twenty-three-item test analyzes the following areas: computation and estimation, probability and statistics, graphical analysis and common functions, and logic/reasoning (For details, see http://serc.carleton.edu/qlra/index.html).

AAC&U’s VALUE (Valid Assessment of Learning in Undergraduate Education) project has published a rubric for assessing quantitative literacy with six criteria: interpretation, representation, calculation, application/analysis, assumptions, and communication (see page 2). Each of these criteria is described in detail, and the performance rating system ranges from the highest level (4 or “capstone”) through mid-range “milestones” (3, 2) to the beginner level (1). The rubric may be downloaded from the web; as with all its VALUE rubrics, AAC&U encourages institutions to modify this one to reflect local emphases. Dingman and Madison (2011) have developed a modified rubric based on AAC&U’s prototype. Grawe et al. (2010) have published a rubric for assessing QR skills within the context of writing assignments.

One concern that faculty may have regarding QR assessment is that it may be perceived as yet another outcome to assess, on top of all of the others, and campuses may be thinking that they need to create a whole new assessment strategy for this outcome. Workload issues are real...
describes several resources for teaching and measuring QR, such as those provided by three national organizations, the Mathematical Association of America, Project Kaleidoscope, and the National Numeracy Network. Other resources are available on the Science Education Resource Center website. These organizations’ websites offer a variety of curricular materials, along with assessment resources. NNN also publishes a national journal, Numeracy, that "supports education at all levels that integrates quantitative skills across disciplines."

This type of teaching has implications for faculty development: not only do faculty members need to be comfortable with the content of QR, but they also need to become skilled in adapting real-world materials to instruction and using more active, less lecture-focused instructional methods. As the writing across the curriculum movement has learned, one of the best ways to help faculty members incorporate QR learning into their courses may be workshops sponsored by the faculty development center. These workshops can help faculty members gain confidence and skills in generating assignments and developing classroom activities for QR in disciplines that do not routinely use mathematics, such as in the arts and humanities. Faculty in these disciplines may also have math anxiety, much as faculty in the sciences and engineering may have anxiety about teaching and grading writing.

CONCLUSION

Hughes-Hallett (2001) asserts that what we need is a partnership among departments to help students achieve QR learning outcomes. She argues that this partnership must involve high schools, community colleges, colleges, and universities. Like the writing across the curriculum programs of the past decade, QR deserves the same institutional attention and focus.

REFERENCES


