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## I. Introduction

We summarize high-level conclusions from a two-day Workshop at the University of Arizona, held in 2019, and dedicated to quantitative intuition (QI) in higher education. A multidisciplinary cross-section of the faculty spanning nine colleges (see About this Workshop) discussed **what QI is, why it matters, and what can be done to support curricular integration across the University**. In this summary report, we review important lessons learned and recommendations derived from those conversations.

A list of resources and readings discussed during the workshop, as well as authors' bios appears at the end of the report. More information about the workshop, organized and hosted by the *Center for University Education Scholarship (CUES)* appears in the Appendix.

## II. Defining QI

Quantitative intuition (QI) may be defined as the ability to rapidly access and use quantitative information *as a habit of mind*, i.e., using data to make a practical decision grounded in empirical evidence. However, QI is difficult to characterize. Is it a well-defined set of skills? Is it an attitude? Does it demand active engagement? What does “quantitative” include or mean? We choose not to answer these questions directly in this workshop and instead adopted the following perspective. QI requires some quantitative knowledge, including how to deploy numbers, graphs, and symbols. QI seems to differ from critical thinking, however. For example, QI solicits a quantitative insight, whereas critical thinking is broader—it is not always quantitative or grounded primarily on insight. Importantly, QI is driven by the question “How might I approach the issue?” with the intent of making a decision and does not necessarily ask for a complete solution. QI may involve estimation, questioning, tinkering, organizing, or pattern recognition. Attitudes that often accompany QI include skepticism, tacit awareness, (dis)comfort, confidence, and curiosity.

**Developing QI among students is an interdisciplinary issue.** While the tools used in applying QI are often taught in mathematics or STEM courses, the insights are developed by integrating across disciplinary perspectives. QI involves being able to glean quantitative insights in a wide variety of contexts. Thus, multidisciplinary involvement is essential to develop robust QI. **The effort to develop QI avoids, if not breaks down, disciplinary silos in service of “lateral thinking”<sup>4</sup>.**

## III. Importance of Quantitative Intuition (QI)

The growing influence of digital data around us and the need for evidence-based decision-making makes QI crucial to both the educational and research mission of the University of Arizona students. For a large number of our students—especially those in STEM disciplines and many social sciences—quantitative skills are essential. However, quantitative judgements permeate our society on a much wider scale, from the mundane (deciding how to allocate a budget across grocery items each week) to the more consequential (interpreting infographics in news articles versus findings in published literature). As governments and private entities experiment with artificial intelligence and technocratic solutions to social problems,

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<sup>4</sup> De Bono, E. *Lateral Thinking: Creativity Step by Step*. Harper Perennial, 2015.

students should understand the basic parameters behind such problems and solutions. In short, we believe that QI is central to a well-informed life in the modern age, regardless of one's future role in society.

#### **IV. Recognizing QI**

Intuition is rarely a natural skill. It is rather something we acquire over time. Therefore, students' QI capabilities can be measured by their readiness to access prior observations and/or knowledge gained over time, and apply these to (1) make sense of new information and (2) derive potential inferences. Situations that foster the development of QI encourage constructing mental and formal models. For example, asking someone which methodology or formula to use given the information available is more likely to promote QI than asking someone to calculate using a formula. When presented with a scenario, a "high-QI" student will form a conceptual picture, whether it be an image based on the physical world, a flow chart, or some other meaningful way of organizing the quantitative information. That first model may not be accurate, but a high-QI student will be able to enhance or expand it. Another manifestation of high-level QI is the ability to respond skeptically to existing solutions. When presented with a claim, QI may invite questioning and "pushback"—perhaps a desire to consult other data sources or models—rather than accepting conclusions without deliberation.

These two examples—critical use of formulas and skepticism—are not the only manifestations of QI; they are merely illustrations. Workshop participants agreed that one can often recognize QI at work when students approach information using these methods. In all its manifestations, QI reveals itself through *active engagement* with quantitative information, whether to make positive observations or normative claims.

#### ***Sparking QI***

##### *The Instructor's Role*

A student's environment and external motivation can spark QI. Workshop participants noted that faculty modeling, i.e., demonstrating enthusiasm and support for QI-based reasoning, is important for fostering a culture of evidence-based thinking. When instructors encourage students to conjecture and guess, even if the first attempt is incorrect, students learn how to update their beliefs and develop comfort with refining ideas through observing data.

The instructor may begin with a "gentle introduction," accessible to all levels of QI, avoiding both, abstraction and symbols. **Allowing students to understand a concept in plain language rather than in symbols can help students develop QI.** Especially at the start, the instructor needs to be careful not to intimidate students. As the semester evolves and trust is established, the instructor must begin to *foster* discomfort, for example, by posing challenging questions that require students to consult new information. **Promoting a risk-free environment where students can ask questions free from (the fear of) judgment, but not free from critical appraisal, is essential for engagement with QI.** Encouraging students to "play" with information, explore different possibilities, generate hypotheses, model, and overcome hesitation about making mistakes, all develop QI. This is because QI involves a metacognitive approach to quantitative problem-solving.

##### *The Students' Role*

Given that developing QI requires more than recognizing it in action, workshop participants proposed instructors should enable students to engage in tasks that foster QI early on. For example, having students take the lead on quantitative interpretation, rather than follow a rote algorithm, engages them with QI. Students can then choose between symbols or plain language eventually realizing the efficiency of symbols for supporting QI, rather than obstructing its development.

Regular student feedback that is grounded on opportunities for model building and "push-back," critically guides instructors as they refine course content and pedagogy. Real-time opportunities for such student feedback may be created through in-class polls (e.g., Top Hat, Socrative) establishing a cycle to sharpen QI through immediate input. Instructors who actively solicit and use this feedback empower

students to think and act experimentally. This mental “playfulness” is at the heart of QI because it represents the ability to reframe information.

The next two sections specifically identify immediate feedback<sup>5</sup> and other known general best practices<sup>6</sup> as enhancing of QI.

### *Course Structure and Design*

Participants also noted that decisions about course structure can spark QI. Establishing learning objectives and identifying eventual academic products at the beginning of a course can motivate students to engage with difficult quantitative issues along the way. An instructor might consider adopting a “backward-designed” class<sup>7</sup>. This approach involves deciding on the objectives first and then inserting the content and pedagogy that best achieve those objectives. In addition, developing a storyline that weaves the course together helps students to recall where they have been and what more needs to be accomplished. Similarly, faculty observed that students are more receptive to problems presented in relatable contexts; they are naturally more curious about problems that stem from their lived experiences. Having former students explain to new students how they use QI is also a great motivator.

The group also shared how associating complex analytical exercises with memorable experiences deepens QI motivation. The more memorable the experiences (e.g., using a particularly relevant context, setting, or technology) the more likely it is that students will convert the short-term to longer-term memory. Ultimately, recollection of experiences deepens one’s reflexive responses, unpacks memories, and promotes QI. Regardless of the type of motivation, the instructor should consider their audience’s interests, background, and feedback to create the most appropriate framework. Since it is not feasible to expect all students to know their future trajectory (whether professional or educational), it is important to strike a balance among relevance, depth, and breadth.

### ***Dampening QI***

Participants also devoted time to factors and circumstances that are counterproductive for bolstering QI. Namely, participants were asked what practices and pedagogies are inconsistent with QI. Although the conversation was robust, we have focused in this report on QI enhancers and mention only in passing QI dampeners. Discussants agreed widely that any classroom approach foregrounding rote memorization is antithetical to QI development. Highly reductive computational procedures—those that, for example, prioritize elements of a formula over the choice among formulae—were considered unhelpful. Failing to see a coherent narrative across the curriculum may also demotivate students. Attempting to cover a large amount of content in a course can overwhelm students with information rather than encouraging them to process smaller amounts more deeply and inventively. A stressful environment led by a dismissive instructor also dampens QI development. Simply stated, the instructor should exhibit and stimulate a growth mindset<sup>8</sup> in the classroom.

## **V. Recommendations**

We propose **establishing and enhancing QI begins with agreement on principles for quantitative reasoning and intuition across fields**. Next, University stakeholders should generate a list of competencies that overlap across fields and are not concentrated on a particular discipline. The University then would provide opportunities for instructors’ growth, leadership, and innovative practice in the QI space. Some

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<sup>5</sup> Holstead, C. [Want to Improve your teaching? Starts with the basics](#). The Chronicle of Higher Education, 2019.

<sup>6</sup> Deslauriers, L., McCarty, L., Miller, K., Callaghan, K., and Kestin, G. (2019). Measuring actual learning versus feeling of learning in response to being actively engaged in the classroom. PNAS. [doi.org/10.1073/pnas.1821936116](https://doi.org/10.1073/pnas.1821936116)

<sup>7</sup> Wiggins, G. and McTighe, G. *Understanding by Design*. Alexandria, VA: Association for Supervision and Curriculum Development, 1998.

<sup>8</sup> Dweck, C. *Mindsets and math/science achievement*. New York: Carnegie Corporation of New York, Institute for Advanced Study, Commission on Mathematics and Science Education, 2008.

form of ongoing QI practice support for instructors might also be useful. For example, quantitative consultants, while not doing most of the direct classroom work, could advise faculty on best practices for communicating QI-enhancing information.

We offer the following four recommendations that emerged from the Workshop.

***Recommendation 1: Emphasize Impressions and Skepticism over Answers***

- Emphasize the art of questioning with healthy skepticism and prioritize estimates rather than complete answers in foundational University courses.
- Utilize different modalities to integrate QI in responses. The instructor should promote the use of gut instincts (i.e., sensing that something is right or wrong) and visualization as methods of thinking quantitatively. Both are fundamental to QI.

***Recommendation 2: Create a QI Framework for further Work***

- Create a framework that characterizing QI as a part of a document that synthesizes evidence on the importance of QI and models for developing it in the field. Such a document could be the starting point for a larger, deeper workshop on QI. The main goal of this second workshop would be to further develop these preliminary ideas about QI, especially gathering input from more departments and developing a vision for the future.

***Recommendation 3: Obtain Input from Different Departments***

- Assess what departments are currently doing with respect to QI. Organize focus groups, or interviews, not just about what departments are doing now, but also on their aspirations. Since QI is interdisciplinary, input from many departments is essential. Every department should have a quantitative plan about how to enhance and develop QI for their discipline.
- Learn what employers of students in various departments tend to expect from graduates seeking jobs in their industries.

***Recommendation 4: Engage in Further Work***

- **Host another, larger workshop** bringing together faculty and employers from more diverse disciplines to inspire QI across fields. The goal of this workshop is to identify where and how types of QI intersect across disciplines, primarily by examining materials currently used to facilitate QI (e.g., visuals, thought experiments, and models).
- **Develop a model for strengthening QI in low-division undergraduate courses, including general education competencies.** (That said, development of QI should not be restricted to general education curricula).
- **Create a toolbox with QI resources** accessible to all educators. Resources would include examples and visualizations.
- **Create a team-based competition across the University** for developing and funding QI-related programs, including the suggested toolbox, workshops, etc. This could incentivize teams of educators from different discipline to create the best possible product.

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## APPENDIX

### About the QI MECha Workshop

In April 2019 the *Center for University Education Scholarship (CUES)* hosted the first Mapping Educational Challenges (MECha) Workshop, on the theme of Quantitative Intuition (QI). The two-day knowledge-generating event, brought together 25 faculty, administrators, and instructors representing nine UA Colleges: Architecture, Law, Eller, Medicine, Public Health, Nursing, Agricultural and Life Sciences, Engineering, and Science.

The goal of the CUES MECha QI workshop was to generate knowledge expanding the scholarship on quantitative intuition, leveraging expertise and interest from UA instructors from various disciplines. More information about the event, including agenda, readings, and the organizing committee, may be found at: [cues.arizona.edu/mecha/](http://cues.arizona.edu/mecha/)

### Author Bios

**Deborah Hughes Hallett** is Professor of Mathematics at the University of Arizona and Adjunct Professor of Public Policy at the Harvard Kennedy School. She organized the Calculus Consortium based at Harvard, which brought together faculty from a wide variety of schools to work on undergraduate curricular issues. She is regularly consulted on the design of curricula and pedagogy for undergraduate mathematics at the national and international level. Her work has been recognized by prizes from Harvard, the University of Arizona, the AAAS, Association for Women in Mathematics, and as national winner Mathematical Association of America (MAA) Award for Distinguished Teaching.

**Hoshin V. Gupta** is a Regents Professor in Hydrology and Atmospheric Sciences at the University of Arizona. Hoshin's interest is in how we build and use "models" and "data" to understand, explore and manage the world around us – processes that are key to science and decision making. Such models span a full range of representations, from perceptual-conceptual mental (involving both qualitative and quantitative intuition), to mathematical/symbolic (formal specification) and computational. Hoshin teaches introductory and advanced courses in Earth Science and Model-Building, and has received the UA Graduate College 2014-15 *Graduate and Professional Education Teaching and Mentoring Award*. He is a *Fellow of the American Geophysical Union*, and is named on the *Clarivate Highly Cited Researchers List*.

**Christopher L. Griffin, Jr.** is a Visiting Professor and Research Scholar at the University of Arizona College of Law. Christopher's research primarily uses randomized control trials to evaluate the effectiveness of innovations in civil and criminal law administration. These studies include tests of a pretrial risk assessment tool, attorney triage in eviction proceedings, and efforts to address the civil legal needs of domestic abuse survivors. Christopher teaches courses in Civil Procedure, Remedies, and Empirical Methods in the Law. He holds a B.S. in International Political Economy, *magna cum laude*, from Georgetown University's School of Foreign Service; an MPhil in Economics from the University of Oxford; and a J.D. from Yale Law School.

### Resources and Readings

In addition to the reference cited in the report, the following articles and media help inform the discussion during the workshop.

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